# A Analytic Queueing Performance of a Combined Input Crosspoint Queued (CICQ) Switch under Bursty Traffic. 

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#### Abstract

This paper considers a $2 \times 3$ Combined Input Crosspoint Queued (CICQ) Switch with an unbalanced grid under bursty traffic. Based on a probabilistic argument, we focus on a Markov Modulated Poisson process (MMPP) at the arrival into the network switch model with a view to analysing and validating its optimum performance measure in terms of a congestion control mechanism suitable for measuring bursty traffic.

Using the Chapman-Komogorov equation, the flow balance equation at departure instants of a queueing model corresponding to the swith architecture is derived. Two system strategies which are partial saturation (PS) model and the full saturation (FS) model are analysed for various system parameters representing MMPP mechanism ${ }^{\mathrm{t}}{ }_{\alpha, \beta}$ ) and arrival ${ }^{\mathrm{t}}{ }_{\lambda}$ ) to three arrival queues and deterministic service time ( $\tau$ ) which leads to 27 runs for both models. Numerical analysis of the algorithms is given, which shows the reliability and efficiency of a model over its counterpart. This system is applicable to the Heavy Occupancy Vehicle (HOV) intervention as practised in the road traffic network. The model is scalable and applicable to more complex systems.


[^0]Keywords: Stationary probabilities; Saturation models; Markov Modulated Poisson Process; Congestion control mechanism; Partial saturation.

## 1 Introduction

This study developed an unbalanced $2 \times 3$ Combined Input Crosspoint Queued (CICQ) switch architecture model with three input queues (IQ) embedded in each of its two virtual output queues VOQ ( A and B ) at the point of entry and resulting three output queues ( OQ ) at the point of exit for the bursty traffic system of packet data traffic. This was with a view to analysing and validating the performance measures in terms of congestion control under a Markov Modulated Poisson Process (MMPP/D/3/RS) regiment. This will compliment the works of $r_{3}$ J on best efforts and priority queuing policies for buffered crossbar switches, ${ }^{r} J$ on Practical Traffic Generation Model for Wireless Networks, and ${ }^{r} 8$ J on Wide-area Traffic: The Failure of Poisson Modeling.

By observing the system at server departure points, relevant flow balance equations at the end of service timeslot were established. This involved $2^{3}$ departure categories leading to $2^{3} \times 2^{3}$ transition probability matrix (TPM). The resulting $2^{3}$ linear equations in $2^{3}$ unknowns were amenable to solutions by the Gauss-Jordan elimination method. Specifically considered for the qualitative and quantitative validation were the Full Saturation (FS) model, for which it was assumed that the system was always occupied, and the Partial Saturation (PS) model, for which it was assumed that only VOQ B was always occupied.
Away from the introduction, the rest of the paper is organised as follows. After describing the model in section 2, the state of the system and the solution of the resulting flow balance equation are given in section 3 . This leads to the main result in section 4: Our experimentation is carried out for the strategies of full saturation model and partial saturation in section 5 . Discussion, conclusion and recommendations round off the paper.


Figure 1: Switch Model of a $2 \times 3$ CICQ Switch Architecture

## 2 Model Description

We consider a crosspoint switch with N input ports and M output ports. Packets, of equal lengths arrive each input port, and each packet is labelled with an output number on which it has to leave the switch.

The switch has three levels of buffering:- each input $i$ maintains for output $j$ a separate Virtual Output Queue of $V O Q_{i}$ of capacity Input Buffer ( $/ B_{i j}$ ); each crosspoint corresponding to input $i$ and output $j$ occupies Crosspoint Queue ( $C Q_{i j}$ ) of capacity Crosspoint Buffer ( $C B_{i j}$ ); each output $\boldsymbol{j}$ maintains a queue Output Queue $\left(O Q_{j}\right)$ of capacity Output Buffer $O B_{j}(i=1,2, \ldots, N ; j=1,2, \ldots, M)$. This
buffering model defines the order packets should be fetched out of the buffer. For a $2 \times 3$ case, this translates into some queueing representation of traffic of two (2) entry points and three (3) output points. We consider the First-In-First-Out (FIFO) discipline under which packets must leave the buffer in the order of their arrivals and a random MMPP selection at service contention point, where the server has a choice of which one of two units to serve at a particular time. The MMPP process is such that a service from VOQ A is followed by its kind with probability $\alpha$ and a service from VOQ B is followed by its kind with probability $\beta$. Along the line of ${ }^{r} 7$, we list below the following general assumptions employed in the development of the queueing model.

1. The switch is a non-symmetric $N \times M$ and each input/output port operates synchronously at the same fixed service time $\tau$ slots, where each time slot is normalized as the time interval for transmitting a cell at the input/output port speed;
2. Each time slot comprises two phases in sequence: the input scheduling phase and the output scheduling phase;
3. The traffic at each input port is i.i.d, with a mean load $\lambda_{1}+\lambda_{2}+\lambda_{3}=\lambda$;
4. At most one cell can arrive at each input port, only at the beginning of a time slot, and at most one cell can depart at each output port, only at the end of a time slot.
5. Both the input and the output arbitrations use the random selection policy, i.e. to select one randomly from all participating candidates of a contention;
6. Each arriving cell is destined to any output port with a probability $\frac{\lambda_{i}}{\lambda}$;

## 3 State of the system

Let the state of the system at time $n$ be denoted by $X_{n}$. We note that $X_{n}$ can be defined by the nature of departure at a moment just after a time slot. This can be
determined by the type of departing unit for a combination of the three respective output ports. The appropriate listing for the states of the system is given as

$$
S=\left\{X_{n}, n=1,2, \ldots \boldsymbol{\}}\right.
$$

or

$$
S_{n}=\left\{A_{1} A_{2} A_{3}, A_{1} A_{2} B_{3}, A_{1} B_{2} A_{3}, A_{1} B_{2} B_{3}, B_{1} A_{2} A_{3}, B_{1} A_{2} B_{3}, B_{1} B_{2} A_{3}, B_{1} B_{2} B_{3} \boldsymbol{\}},\right.
$$

or

$$
S_{n}=\left\{S_{1}, S_{2}, S_{3}, S_{4}, S_{5}, S_{6}, S_{7}, S_{8} \boldsymbol{\}}\right.
$$

in the obvious lexicographical order.
Typically,

$$
S_{n}=\left(Y_{1}, Y_{2}, Y_{3}\right)
$$

where $Y_{i}=A_{i}$ or $B_{i}$ depending on if the departing unit is from VOQ A or from VOQ B. Thus; for example

$$
S_{1}=A_{1}, A_{2}, A_{3}
$$

connotes that the departing unit in output port 1,2 and 3 are all from VOQ A. The states $X_{n}$ form an embedded Markov chain such that the steady state solutions are connected by ${ }^{r}{ }_{9}$

$$
\begin{equation*}
S_{j}={ }_{i=1}^{8} p_{i j} S_{i} \tag{1}
\end{equation*}
$$

$j=1,2, \ldots 8$ and $p_{i j}$ is the transition probability of moving from state $S_{i}$ to state $S_{j}$, from one departing moment to the next departing moment.
Equation(1) can be expressed for each $j$.

$$
\begin{equation*}
-p_{i j} S_{1}-p_{2 j} S_{2}+p_{3 j} S_{3}+. .+p_{j j}\left(S_{j}-S_{i}\right)+\ldots+p_{8 j} S_{j}=0 \tag{2}
\end{equation*}
$$

The system of equation (1) can be expressed in expanded form as

$$
\begin{aligned}
& S_{1}\left(1-p_{11}\right)-p_{21} S_{2}-p_{31} S_{3}-\ldots-p_{71} S_{7}-p_{81} S_{8}=0 \\
& -S_{1} p_{12}+\left(1-p_{22}\right) S_{2}-p_{32} S_{3}-\ldots-p_{72} S_{7}-p_{82} S_{8}=0
\end{aligned}
$$

$$
\begin{equation*}
-S_{1} p_{13}-p_{23} S_{2}+\left(1-p_{33}\right) S_{3}-\ldots-p_{73} S_{7}-p_{83} S_{8}=0 \tag{3}
\end{equation*}
$$

$$
S_{1} p_{17}-p_{27} S_{2}-p_{37} S_{3}-\ldots+\left(1-p_{77}\right) S_{7}-p_{87} S_{8}=0
$$

The system of equations in (3) can be solved for probability vectors $S_{j}$, with the condition

$$
{ }_{j=1}^{8} S_{j}=1
$$

This can be put in matrix form as


Subject to knowledge of the conditional probability $p_{i j}, i, j=1,2, \ldots 8$, the matrix system in Equation (1) can be solved algebraically using the gaussian elimination method ${ }^{r} 2$ J. It can also be solved with the use of the common interchanges. The alternative is to start with an initial transition probability matrix (TPM) $P$ and then take the limit of the power $P^{n}$ as $n \rightarrow \infty$. This leads to the limiting vector $W$ of common probability vectors ( $W_{1}, W_{2}, W_{3}, W_{4}, W_{5}, W_{6}, W_{7}, W_{8}$ )

## 4 Main results

Based on the above, two other expressions of the stationary probability distribution are derived, which lead to two alternative algorithms for computing stationary probabilities. Assuming some form of independent for the 3 output ports, the probability of the 3 VOQ $A$ ports being empty is given by

$$
\begin{equation*}
\left(1-\frac{\lambda_{11} T}{\pi_{1}}\right)\left(1-\frac{\lambda_{12} T}{\pi_{2}}\right)\left(1-\frac{\lambda_{13} T}{\pi_{3}}\right) \tag{6}
\end{equation*}
$$

which by stability condition of the system is strictly positive. The condition thus has to be satisfied by the choices of arrival rate $\left(\lambda_{11}, \lambda_{12}, \lambda_{13}\right)$, fixed service time
rate ( $\tau$ ), and probability rate ( $\alpha_{1}, \alpha_{2}, \alpha_{3}, \beta_{1}, \beta_{2}$ and $\beta_{3}$ ) for stability.
We observe that $\pi_{\text {}}$ is steady state probability that Markov Modulated mechanism in output port $/$ is serving from VOQ $A$ is given by

$$
\begin{equation*}
\pi=\frac{1-\beta}{2-\alpha-\beta} \tag{7}
\end{equation*}
$$

Probability of no arrival into input port $i$ of VOQ $A$ during the fixed service time $\tau, I=1,2,3$ is $e^{-\lambda_{1} \tau}$;
Probability of at least one arrival into input port / of VOQ A of VOQ A during a fixed service time $\tau$ is $1-e^{-\lambda_{/ I}}$;
the probability that input port $/$ is empty is given as

$$
\begin{equation*}
p_{o l}=1-\frac{\lambda_{I} T}{\pi_{l}} \tag{8}
\end{equation*}
$$

It follows then that the probability $p_{i j}$ of moving from a state $S_{i}$ into a state $S_{j}$ is given by


Equation (9) and (10) followed by allowing independent input and also conditioning on the probability that VOQ $A$ is occupied or unoccupied at a preceding state. It is clear that Equation(9) determines the entries of the TPM for every of the $8 \times 8$ combinations of $S_{j}$ and $S_{j}$ in terms of the parameters $\lambda_{l}, \tau, \alpha_{l}, \beta_{l}$ and of course, $\pi_{l}$ and $p_{o l}$, for $I=1,2,3$.
It is observed to note that the partially saturated models reduces to the TPM for the full saturated model if $\eta_{I}$ and $p_{0 I}=1$ for all input ports. For Equation(10) clearly indicates that $\alpha_{I}=0, \alpha_{I}=1, \beta_{I}=0, \beta_{I}=1$ for a meaningful non-absorbing Markov Modulated mechanism. With the term in (10) determined, it is a question of time to solve for the probability vector $\left(S_{1}, S_{2}, S_{3}, S_{4}, S_{5}, S_{6}, S_{7}, S_{8}\right)$.

## 5 Experimentation

In the case of the fully saturated model, we solve for the probability vectors for 27 RUNS determined by
$\alpha_{1}=0.25,0.50,0.75, \quad \beta_{1}$ (fixed) $=0.75$
$\alpha_{2}=0.10,0.25,0.50, \quad \beta_{2}($ fixed $)=0.1$
$\alpha_{3}=0.25,0.50,0.75, \quad \beta_{3}$ (fixed) $=0.25$
Similarly, for the partially saturated models, we solve for the probability vectors for 27 subsystems determined by
$\alpha_{1}=0.25,0.50,0.75, \quad \lambda_{1}=1$ (fixed) $\quad \beta_{1}($ fixed $)=0.75$
$\alpha_{2}=0.10,0.25,0.50, \quad \lambda_{2}=2$ (fixed) $\quad \beta_{2}($ fixed $)=0.10$
$\alpha_{3}=0.25,0.50,0.75, \quad \lambda_{3}=3$ (fixed) $\quad \beta_{3}$ (fixed) $=0.25$
It is thus possible to compare the transition vectors from RUN to RUN for the same model and from fully saturated model to partially saturated model for the same system.

Table 1 shows the (8) eight steady state probability vectors for both the Full saturated $(F)$ strategies and the Partial saturated strategies for all 27 RUN cases with the Euclidean distance between them in the last column. The Euclidean is a measure of the extent by which the relative probabilities of each state differs one strategy to the other.

Table 2 shows the 27 RUNS and the Euclidean distance (gap) between the full and the partial saturation model for various service time $(\tau=) 0.1,0.09,0.08,0.07$, $0.06,0.05,0.04,0.03,0.02$ and 0.01 . On the right, for ease of reference, are the varying $\alpha$ parameters as they define the RUNS.
Table 1 and Table 2 indicate as follows:

1. for fixed $\tau, \alpha_{2}$ and $\alpha_{3}$, the distance measure generally increases for increasing values of $\alpha_{1}$ in all nine (9) cases considered.
2. for fixed $\tau, \alpha_{1}$ and $\alpha_{2}$, the distance measure appears to remain of the same order and not change significantly for the cases considered as $\alpha_{3}$ increases.

Table 1: Steady state probability vector ( $W$ ) for the Full Saturated model for RUN
1 to RUN 27 and the Euclidean distance between both strategies

| RUN | strategy | $W_{1}$ | $W_{2}$ | $W_{3}$ | $W_{4}$ | $W_{5}$ | $W_{6}$ | $W_{7}$ | $W_{8}$ | Distance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 001 | F | 0.05000 | 0.15000 | 0.05000 | 0.15000 | 0.075000 | 0.225000 | 0.075000 | 0.225000 | 0.2801 |
|  | P | 0.01468270 | 0.02292594 | 0.03043375 | 0.04757821 | 0.1138036 | 0.1776959 | 0.2358880 | 0.3683299 |  |
| 002 | F | 0.066667 | 0.133333 | 0.0666667 | 0.133333 | 0.100000 | 0.200000 | 0.100000 | 0.200000 | 0.2644 |
|  | P | 0.01521997 | 0.02208569 | 0.03154820 | 0.04580266 | 0.1179679 | 0.1711832 | 0.2445259 | 0.3548328 |  |
| 003 | F | 0.105263 | 0.105263 | 0.0947367 | 0.0947367 | 0.157895 | 0.157895 | 0.142105 | 0.142105 | 0.2775 |
|  | P | 0.01549391 | 0.02169459 | 0.03211643 | 0.04496945 | 0.1200913 | 0.1681519 | 0.2489302 | 0.3485523 |  |
| 004 | F | 0.0545455 | 0.163636 | 0.0454545 | 0.136364 | 0.0818182 | 0.245455 | 0.0681818 | 0.204545 | 0.2973 |
|  | P | 0.01453426 | 0.02269416 | 0.03012607 | 0.04709720 | 0.1126531 | 0.1758994 | 0.2335032 | 0.3646061 |  |
| 005 | F | 0.0727273 | 0.145455 | 0.0606061 | 0.121212 | 0.109091 | 0.218182 | 0.0909091 | 0.181818 | 0.2931 |
|  | P | 0.01554377 | 0.02458716 | 0.03376791 | 0.04901987 | 0.1313289 | 0.1905718 | 0.2617306 | 0.3597987 |  |
| 006 | F | 0.109091 | 0.109091 | 0.0909091 | 0.0909091 | 0.163636 | 0.163636 | 0.136364 | 0.136364 | 0.2829 |
|  | P | 0.01571344 | 0.02200197 | 0.03189690 | 0.04466207 | 0.1217928 | 0.1705344 | 0.2472286 | 0.3461697 |  |
| 007 | F | 0.080350 | 0.241071 | 0.0446429 | 0.133929 | 0.080350 | 0.241071 | 0.0446429 | 0.133929 | 0.3889 |
|  | P | 0.01500238 | 0.02342549 | 0.02989858 | 0.04672910 | 0.1162815 | 0.1815678 | 0.2317399 | 0.3618530 |  |
| 008 | F | 0.0857143 | 0.171429 | 0.047619 | 0.0952381 | 0.128571 | 0.257143 | 0.0714286 | 0.107143 | 0.3651 |
|  | P | 0.01561077 | 0.02458716 | 0.03376791 | 0.04901987 | 0.1313289 | 0.1905718 | 0.2617306 | 0.3597987 |  |
| 009 | F | 0.187500 | 0.187500 | 0.187500 | 0.187500 | 0.062500 | 0.062500 | 0.062500 | 0.062500 | 0.3463 |
|  | P | 0.01590750 | 0.02227369 | 0.03170284 | 0.04439035 | 0.1232969 | 0.1726405 | 0.2457245 | 0.3440637 |  |
| 010 | F | 0.062500 | 0.187500 | 0.062500 | 0.187500 | 0.062500 | 0.187500 | 0.062500 | 0.187500 | 0.3465 |
|  | P | 0.01500320 | 0.02342493 | 0.03109617 | 0.04864142 | 0.1336705 | 0.2087149 | 0.2770653 | 0.4326256 |  |
| 011 | F | 0.08333 | 0.166667 | 0.08333 | 0.166667 | 0.08333 | 0.166667 | 0.08333 | 0.166667 | 0.3333 |
|  | P | 0.01523269 | 0.02065280 | 0.02950114 | 0.04284110 | 0.1268167 | 0.1840234 | 0.2628673 | 0.3814477 |  |
| 012 | F | 0.125000 | 0.125000 | 0.125000 | 0.125000 | 0.125000 | 0.125000 | 0.125000 | 0.125000 | 0.332500 |
|  | P | 0.01568074 | 0.01915578 | 0.02835801 | 0.03970691 | 0.1219045 | 0.1706907 | 0.2526887 | 0.3538149 |  |
| 013 | F | 0.0681818 | 0.204545 | 0.0568182 | 0.170455 | 0.0681818 | 0.204545 | 0.0568182 | 0.170455 | 0.358100 |
|  | P | 0.01301868 | 0.02032678 | 0.02642560 | 0.04132943 | 0.1159917 | 0.1811126 | 0.2354516 | 0.3676476 |  |
| 014 | F | 0.0909091 | 0.181818 | 0.0757576 | 0.151515 | 0.0909091 | 0.181818 | 0.0757576 | 0.151515 | 0.336500 |
|  | P | 0.01339442 | 0.02233886 | 0.03124988 | 0.04536766 | 0.1193201 | 0.1731455 | 0.2422137 | 0.3514775 |  |
| 015 | F | 0.136364 | 0.136364 | 0.113636 | 0.113636 | 0.136364 | 0.136364 | 0.113636 | 0.113636 | 0.344100 |
|  | P | 0.01387432 | 0.01942684 | 0.02816441 | 0.03943584 | 0.1236294 | 0.1731060 | 0.2509636 | 0.3513995 |  |
| 016 | F | 0.0803575 | 0.241071 | 0.0446429 | 0.133929 | 0.0803571 | 0.241071 | 0.0446429 | 0.133929 | 0.392400 |
|  | P | 0.014538366 | 0.02270057 | 0.03013458 | 0.04711049 | 0.1126849 | 0.1759491 | 0.2335691 | 0.3647090 |  |
| 017 | F | 0.107143 | 0.214286 | 0.0595238 | 0.119048 | 0.107143 | 0.214286 | 0.059538 | 0.119048 | 0.378800 |
|  | P | 0.01454001 | 0.02270313 | 0.03013798 | 0.0471158 | 0.1126976 | 0.1759689 | 0.2335955 | 0.3647502 |  |
| 018 | F | 0.160714 | 0.160714 | 0.0892857 | 0.0892857 | 0.160714 | 0.160714 | 0.0892857 | 0.0892857 | 0.376900 |
|  | P | 0.01504592 | 0.01966711 | 0.02799281 | 0.03919556 | 0.1251585 | 0.1752470 | 0.2494345 | 0.3492585 |  |
| 019 | F | 0.093750 | 0.281250 | 0.093750 | 0.281250 | 0.031250 | 0.093750 | 0.031250 | 0.093750 | 0.510400 |
|  | P | 0.01054247 | 0.02270698 | 0.03014308 | 0.04712379 | 0.1127167 | 0.1759988 | 0.2336350 | 0.3648120 |  |
| 020 | F | 0.125000 | 0.250000 | 0.125000 | 0.125000 | 0.250000 | 0.083333 | 0.0416667 | 0.083333 | 0.485700 |
|  | P | 0.01064233 | 0.01544285 | 0.02205902 | 0.03203372 | 0.1222281 | 0.1773652 | 0.2533568 | 0.3636475 |  |
| 021 | F | 0.187500 | 0.187500 | 0.187500 | 0.187500 | 0.062500 | 0.062500 | 0.062500 | 0.062500 | 0.507000 |
|  | P | 0.01085904 | 0.01520484 | 0.02250907 | 0.03151722 | 0.1247261 | 0.1746416 | 0.2585375 | 0.36200449 |  |
| 022 | F | 0.102273 | 0.306818 | 0.0852273 | 0.255682 | 0.0340909 | 0.102273 | 0.084091 | 0.0852273 | 0.497100 |
|  | P | 0.01454247 | 0.02270698 | 0.03014308 | 0.04712379 | 0.1127167 | 0.1759988 | 0.2336350 | 0.3648120 |  |
| 023 | F | 0.121212 | 0.242424 | 0.10101 | 0.20202 | 0.0606061 | 0.121212 | 0.0505051 | 0.10101 | 0.441700 |
|  | P | 0.01455068 | 0.02271980 | 0.03016010 | 0.04715040 | 0.1127804 | 0.1760981 | 0.2337670 | 0.3600180 |  |
| 024 | F | 0.181818 | 0.181818 | 0.151515 | 0.151515 | 0.0909091 | 0.0909091 | 0.0757576 | 0.0757576 | 0.438700 |
|  | P | 0.01455068 | 0.02271980 | 0.03016010 | 0.04715040 | 0.11278040 | 0.1760981 | 0.2337669 | 0.3600180 |  |
| 025 | F | 0.107143 | 0.321429 | 0.0595238 | 0.178571 | 0.0535714 | 0.160714 | 0.0297619 | 0.0892857 | 0.496500 |
|  | P | 0.01049391 | 0.02169459 | 0.03211643 | 0.04496945 | 0.1200913 | 0.1681519 | 0.2489302 | 0.3585522 |  |
| 026 | F | 0.142857 | 0.285714 | 0.0793651 | 0.15873 | 0.0714286 | 0.142857 | 0.0396825 | 0.0793651 | 0.476500 |
|  | P | 0.01093915 | 0.02169459 | 0.032116430 | 0.04496945 | 0.1200913 | 0.1681519 | 0.2489302 | 0.3585522 |  |
| 027 | F | 0.214286 | 0.214286 | 0.119048 | 0.119048 | 0.107143 | 0.107143 | 0.0595238 | 0.0595238 | 0.480100 |
|  | P | 0.01114891 | 0.01561071 | 0.02221920 | 0.03111135 | 0.1280555 | 0.1793034 | 0.2552081 | 0.3573427 |  |

Table 2: Euclidean Distance between the Full Saturation and the Partial Saturation
Strategies with Varying Service time $\tau$ for RUN 1 to RUN 27

| RUNS | $\boldsymbol{T}=0.1$ | $\boldsymbol{T}=0.09$ | $\boldsymbol{T}=0.08$ | $\boldsymbol{T}=0.07$ | $\boldsymbol{T}=0.06$ | $\boldsymbol{T}=0.05$ | $\boldsymbol{T}=0.04$ | $\boldsymbol{T}=0.03$ | $\boldsymbol{T}=0.02$ | $\boldsymbol{T}=0.01$ | $\alpha_{1}$ | $\alpha_{2}$ | $\alpha_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 001 | 0.2801 | 0.2924 | 0.3129 | 0.3381 | 0.3710 | 0.4127 | 0.4650 | 0.5303 | 0.6120 | 0.7146 | 0.25 | 0.10 | 0.25 |
| 002 | 0.2644 | 0.2822 | 0.3066 | 0.3371 | 0.3760 | 0.4227 | 0.4803 | 0.5500 | 0.6346 | 0.7373 | 0.25 | 0.10 | 0.50 |
| 003 | 0.2775 | 0.3062 | 0.3402 | 0.3800 | 0.4260 | 0.4802 | 0.5427 | 0.6155 | 0.7007 | 0.8010 | 0.25 | 0.10 | 0.75 |
| 004 | 0.2973 | 0.3119 | 0.3339 | 0.3618 | 0.3950 | 0.4399 | 0.4933 | 0.5592 | 0.6507 | 0.7421 | 0.25 | 0.25 | 0.25 |
| 005 | 0.2931 | 0.3012 | 0.3272 | 0.3596 | 0.3990 | 0.4477 | 0.5060 | 0.5761 | 0.6603 | 0.7617 | 0.25 | 0.25 | 0.50 |
| 006 | 0.2829 | 0.3125 | 0.3474 | 0.3881 | 0.4350 | 0.4898 | 0.5529 | 0.6260 | 0.7110 | 0.8101 | 0.25 | 0.25 | 0.75 |
| 007 | 0.3889 | 0.4078 | 0.4330 | 0.4622 | 0.4980 | 0.5413 | 0.5937 | 0.6575 | 0.7354 | 0.8310 | 0.25 | 0.50 | 0.25 |
| 008 | 0.3651 | 0.3776 | 0.4083 | 0.4443 | 0.4871 | 0.5377 | 0.5973 | 0.6675 | 0.7506 | 0.8490 | 0.25 | 0.50 | 0.50 |
| 009 | 0.4798 | 0.5010 | 0.5261 | 0.5576 | 0.5946 | 0.6387 | 0.6911 | 0.7532 | 0.8266 | 0.9133 | 0.25 | 0.50 | 0.75 |
| 010 | 0.3465 | 0.3616 | 0.3812 | 0.4057 | 0.4368 | 0.4757 | 0.5246 | 0.5860 | 0.6631 | 0.7604 | 0.50 | 0.10 | 0.25 |
| 011 | 0.3333 | 0.3511 | 0.3738 | 0.4024 | 0.4378 | 0.4816 | 0.5351 | 0.6005 | 0.6803 | 0.7778 | 0.50 | 0.10 | 0.50 |
| 012 | 0.3325 | 0.3584 | 0.3887 | 0.4247 | 0.4672 | 0.5171 | 0.5757 | 0.6445 | 0.7255 | 0.8213 | 0.50 | 0.10 | 0.75 |
| 013 | 0.3581 | 0.3753 | 0.3967 | 0.4231 | 0.4559 | 0.4964 | 0.5467 | 0.6089 | 0.6862 | 0.7826 | 0.50 | 0.25 | 0.25 |
| 014 | 0.3365 | 0.3645 | 0.3888 | 0.4189 | 0.4558 | 0.5008 | 0.5514 | 0.6214 | 0.7012 | 0.7977 | 0.50 | 0.25 | 0.50 |
| 015 | 0.3441 | 0.3704 | 0.4018 | 0.4387 | 0.4821 | 0.5328 | 0.5920 | 0.6611 | 0.7418 | 0.8366 | 0.50 | 0.25 | 0.75 |
| 016 | 0.3924 | 0.4157 | 0.4409 | 0.4697 | 0.5052 | 0.5480 | 0.5997 | 0.6626 | 0.7393 | 0.8333 | 0.50 | 0.50 | 0.25 |
| 017 | 0.3788 | 0.4032 | 0.4306 | 0.4629 | 0.5020 | 0.5486 | 0.6042 | 0.6703 | 0.7491 | 0.8430 | 0.50 | 0.50 | 0.50 |
| 018 | 0.3769 | 0.4055 | 0.4385 | 0.4768 | 0.5213 | 0.5727 | 0.6321 | 0.7007 | 0.7801 | 0.8718 | 0.50 | 0.50 | 0.75 |
| 019 | 0.5104 | 0.5486 | 0.5562 | 0.5772 | 0.6140 | 0.6470 | 0.6884 | 0.7409 | 0.8077 | 0.8933 | 0.75 | 0.10 | 0.25 |
| 020 | 0.4857 | 0.5101 | 0.5359 | 0.5651 | 0.5998 | 0.6406 | 0.6891 | 0.7474 | 0.8180 | 0.8994 | 0.75 | 0.10 | 0.50 |
| 021 | 0.5070 | 0.5259 | 0.5488 | 0.5766 | 0.6101 | 0.6504 | 0.6990 | 0.7574 | 0.8277 | 0.9126 | 0.75 | 0.10 | 0.75 |
| 022 | 0.4971 | 0.5361 | 0.5562 | 0.5795 | 0.6089 | 0.6452 | 0.6900 | 0.7460 | 0.8160 | 0.9043 | 0.75 | 0.25 | 0.25 |
| 023 | 0.4417 | 0.4823 | 0.5036 | 0.5292 | 0.5611 | 0.6004 | 0.6484 | 0.7020 | 0.7796 | 0.8680 | 0.75 | 0.25 | 0.50 |
| 024 | 0.4387 | 0.4794 | 0.5050 | 0.5357 | 0.5724 | 0.6162 | 0.6683 | 0.7301 | 0.8034 | 0.8906 | 0.75 | 0.25 | 0.75 |
| 025 | 0.4965 | 0.5230 | 0.5448 | 0.5698 | 0.6007 | 0.6383 | 0.6843 | 0.7407 | 0.8104 | 0.8968 | 0.75 | 0.50 | 0.25 |
| 026 | 0.4765 | 0.5076 | 0.5309 | 0.5586 | 0.5924 | 0.6332 | 0.6825 | 0.7419 | 0.8136 | 0.9000 | 0.75 | 0.50 | 0.50 |
| 027 | 0.4801 | 0.5028 | 0.5298 | 0.5620 | 0.5999 | 0.6447 | 0.6974 | 0.7592 | 0.8316 | 0.9164 | 0.75 | 0.50 | 0.75 |

3. for fixed $\tau, \alpha_{1}$ and $\alpha_{3}$ the distance measure is partly of the same order and partly depends on relative values of $\alpha_{1}$ ansd $\alpha_{3}$ as $\alpha_{2}$ increases.
4. The gap in probability vector between the full and partial saturation model decreases as service time increases. This is to say that as the rate of service becomes slower the gap between the two strategies becomes narrower.

Any introduction of such prediction and traffic congestion increases like the HOV intervention must therefore be accompanied by the removal of any impediment to the free flow of service to make the use of the priority lane meaningful.


Figure 2: A Scatterplot of $T^{\mathrm{t}}=0.01,0.02, \ldots 0.1^{\text {) }}$ vs RUNS

## 6 Conclusion

Overall, the results presented in this paper indicates that the performance of a CICQ switch in terms of traffic flow and congestion control exhibits behaviour that is not counter to intuition and would not seem otherwise in a real life application with an accurate analysis of well defined system parameters. Hence our work provides a plausibility to investigate the behaviour of the CICQ switch network under more realistic assumptions regarding the traffic sources, system parameters and service time, and presents a forward step in understanding decongestion of traffic in terms of network performance. Again, the performance curve shown in Figure

2 establish that the distance measure in the environment depends strongly on the interaction among scheduling and load balancing algorithms, the routing probabilities and the service time parameters.
The analysis also suggest that the allocation scheme used in our study can be successful with the provision of additional capacity in terms of increasing the number of routes and service channels.
This work is specifically for a $2 \times 3$ model with MMBP mechanism and hence can be extended for a possible number of $M$, where $M=N+k$, for $k=2,3,4, \ldots$.

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